**Fattest path.** Given an edge-weighted digraph and two vertices s*s* and *t*, design an *E*log*E* algorithm to find a fattest path from s*s* to *t*. The *bottleneck capacity* of a path is the minimum weight of an edge on the path. A *fattest path* is a path such that no other path has a higher bottleneck capacity.

**Perfect matchings in k-regular bipartite graphs.** Suppose that there are *n* men and *n* women at a dance and that each man knows exactly *k* women and each woman knows exactly *k* men (and relationships are mutual). Show that it is always possible to arrange a dance so that each man and woman are matched with someone they know.

**Maximum weight closure problem.** A subset of vertices *S* in a digraph is *closed* if there are no edges pointing from *S* to a vertex outside *S*. Given a digraph with weights (positive or negative) on the *vertices*, find a closed subset of vertices of maximum total weight.